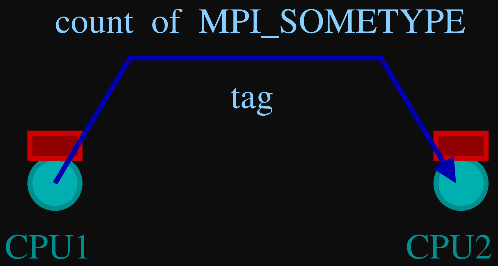
Distributed Parallel Programming with MPI - part 2

Ramses van Zon

PHY1610 Winter 2024

Communication patterns in MPI

Pairwise communication

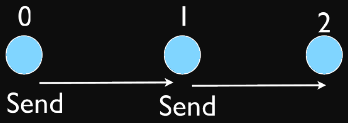


```
$ git clone /scinet/course/phy1610/mpi
$ cd mpi
$ source setup
$ mpicxx -O2 -std=c++17 -march=native -o firstmessage firstmessage.cpp
$ # or: make firstmessage
$ mpirun -n 2 firstmessage
Received 111.000000 on 1
Sent 111.000000 from 0
```

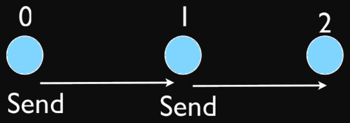
Pairwise communication

```cpp
// firstmessage.cpp
#include <iostream>
#include <string>
#include <mpi.h>
int main(int argc, char **argv) {
    int rank, size;
    double msgsent, msgrcvd;
    MPI_Status rstatus;
    MPI_Init(&argc, &argv);
    MPI_Comm_rank(MPI_COMM_WORLD, &rank);
    MPI_Comm_size(MPI_COMM_WORLD, &size);
    msgsent = 111.;
    msgrcvd = -999.;
    if (rank == 0) {
        MPI_Ssend(&msgsent, 1, MPI_DOUBLE, 1, 746, MPI_COMM_WORLD);
        std::cout << "Sent " + std::to_string(msgsent) + " from " + std::to_string(rank) + "\n";
    }
    if (rank == 1) {
        MPI_Recv(&msgrcvd, 1, MPI_DOUBLE, 0, 746, MPI_COMM_WORLD, &rstatus);
        std::cout << "Received " + std::to_string(msgrcvd) + " on " + std::to_string(rank) + "\n";
    }
    MPI_Finalize();
}
```

Send a message to the right



Send a message to the right



Helpful specials

- Special Source/Destination: `MPI_PROC_NULL`

  `MPI_PROC_NULL` basically ignores the relevant operation; can lead to cleaner code.

- Special Source `MPI_ANY_SOURCE`

  `MPI_ANY_SOURCE` is a wildcard; matches any source when receiving.

- Special Status `MPI_STATUS_IGNORE`

  Use `MPI_STATUS_IGNORE` if you do not want to capture the status in a receive.

Send a message to the right

```cpp
// secondmessage.cpp
#include <iostream>
#include <string>
#include <mpi.h>
int main()
{
    int rank, size, left, right;
    double  msgsent, msgrcvd;
    MPI_Init(nullptr, nullptr);
    MPI_Comm_rank(MPI_COMM_WORLD, &rank);
    MPI_Comm_size(MPI_COMM_WORLD, &size);
    left = rank - 1;
    if (left < 0) left = MPI_PROC_NULL;
    right = rank + 1;
    if (right >= size) right = MPI_PROC_NULL;
    msgsent = rank*rank;
    msgrcvd = -999.;
    MPI_Ssend(&msgsent, 1, MPI_DOUBLE, right, 746, MPI_COMM_WORLD);
    MPI_Recv(&msgrcvd, 1, MPI_DOUBLE, left, 746, MPI_COMM_WORLD, MPI_STATUS_IGNORE);
    std::cout << std::to_string(rank) + ": Sent " + std::to_string(msgsent)
                + " and got " + std::to_string(msgrcvd) + "\n";
    MPI_Finalize();
}
```

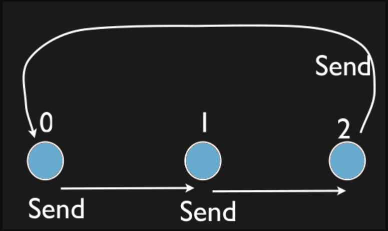
MPI: Send Right, Receive Left

```
$ make secondmessage
$ mpirun -n 3 ./secondmessage
2: Sent 4.000000 and got 1.000000
0: Sent 0.000000 and got -999.000000
1: Sent 1.000000 and got 0.000000
$
```

```
$ mpirun -n 6 ./secondmessage
4: Sent 16.000000 and got 9.000000
5: Sent 25.000000 and got 16.000000
0: Sent 0.000000 and got -999.000000
1: Sent 1.000000 and got 0.000000
2: Sent 4.000000 and got 1.000000
3: Sent 9.000000 and got 4.000000
```

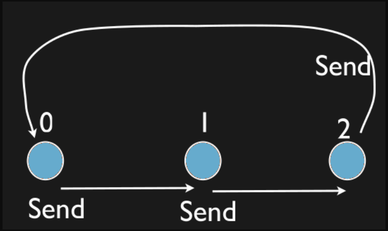
MPI: Send Right, Receive Left with Periodic BCs

Periodic Boundary Conditions:



MPI: Send Right, Receive Left with Periodic BCs

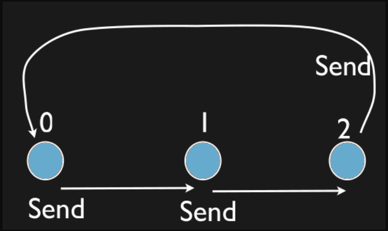
Periodic Boundary Conditions:



```
...
left = rank - 1;
if (left < 0) left = size-1; // Periodic BC
right = rank + 1;
if (right >= size) right =0; // Periodic BC
msgsent = rank*rank;
msgrcvd = -999.;
...
```

MPI: Send Right, Receive Left with Periodic BCs

Periodic Boundary Conditions:



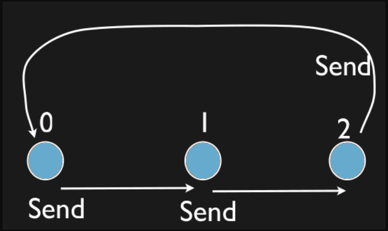
```
    ...
    left = rank - 1;
    if (left < 0) left = size-1; // Periodic BC
    right = rank + 1;
    if (right >= size) right =0; // Periodic BC
    msgsent = rank*rank;
    msgrcvd = -999.;
    ...
```

```
$ make thirdmessage
$ mpirun -n 3 ./thirdmessage

_
```

MPI: Send Right, Receive Left with Periodic BCs

Periodic Boundary Conditions:



```
    ...
    left = rank - 1;
    if (left < 0) left = size-1; // Periodic BC
    right = rank + 1;
    if (right >= size) right =0; // Periodic BC
    msgsent = rank*rank;
    msgrcvd = -999.;
    ...
```

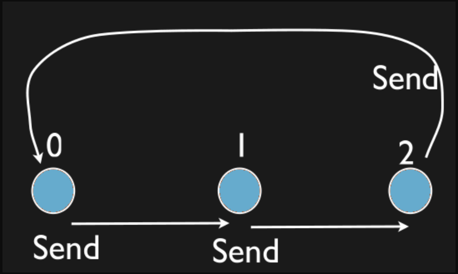
```
$ make thirdmessage
$ mpirun -n 3 ./thirdmessage
_
```

Program hangs!

Deadlock!

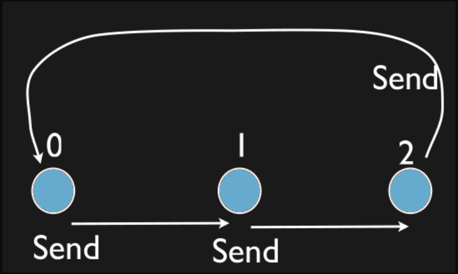
Deadlock!

- A classic parallel bug.
- Occurs when a cycle of tasks are waiting for the others to finish.
- Whenever you see a closed cycle, you likely have (or risk) a deadlock.
- Here, all processes are waiting for the Ssend to complete, but no one is receiving.



Deadlock!

- A classic parallel bug.
- Occurs when a cycle of tasks are waiting for the others to finish.
- Whenever you see a closed cycle, you likely have (or risk) a deadlock.
- Here, all processes are waiting for the Ssend to complete, but no one is receiving.



Sends and receives must be paired when sending!

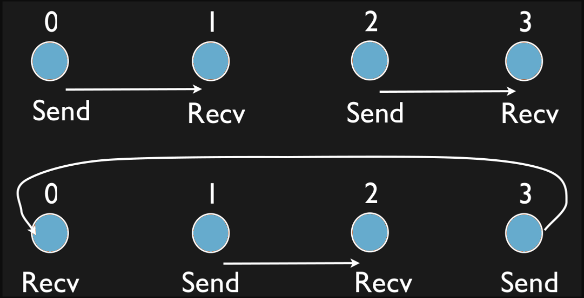
How do we fix the deadlock?

How could we fix the deadlock?

How do we fix the deadlock?

How could we fix the deadlock?

Even-odd solution



- First: evens send, odds receive
- Then: odds send, evens receive
- Will this work with an odd number of processes? How about 2? 1?

MPI: Send Right, Recv Left with Periodic BCs - fixed

```
    ...
    if ((rank % 2) == 0) {
        MPI_Ssend(&msgsent, 1, MPI_DOUBLE, right, 746, MPI_COMM_WORLD);
        MPI_Recv(&msgrcvd, 1, MPI_DOUBLE, left, 746, MPI_COMM_WORLD, MPI_STATUS_IGNORE);
    } else {
        MPI_Recv(&msgrcvd, 1, MPI_DOUBLE, left, 746, MPI_COMM_WORLD, MPI_STATUS_IGNORE);
        MPI_Ssend(&msgsent, 1, MPI_DOUBLE, right, 746, MPI_COMM_WORLD);
    }
    ...
```

```
$ make fourthmessage
$ mpirun -n 5 ./fourthmessage
1: Sent 1.000000 and got 0.000000
2: Sent 4.000000 and got 1.000000
3: Sent 9.000000 and got 4.000000
4: Sent 16.000000 and got 9.000000
0: Sent 0.000000 and got 16.000000
```

MPI: Sendrecv

This kind of exchange is so common and always runs the risk of deadlock, so the MPI standard has a function for that to deal with this scenario.

MPI: Sendrecv

This kind of exchange is so common and always runs the risk of deadlock, so the MPI standard has a function for that to deal with this scenario.

```
MPI_Sendrecv(sendptr, count, MPI_TYPE, destination, tag,
             recvptr, count, MPI_TYPE, source, tag, Communicator, MPI_Status)
```

MPI: Sendrecv

This kind of exchange is so common and always runs the risk of deadlock, so the MPI standard has a function for that to deal with this scenario.

```
MPI_Sendrecv(sendptr, count, MPI_TYPE, destination, tag,
             recvptr, count, MPI_TYPE, source, tag, Communicator, MPI_Status)
```

- A blocking send and receive built together.
- Lets them happen simultaneously.
- Can automatically pair send/recvs.

Send Right, Receive Left with Periodic BCs - Sendrecv

Code

```
    ...
    MPI_Sendrecv(&msgsent, 1, MPI_DOUBLE, right, 746,
                 &msgrcvd, 1, MPI_DOUBLE, left, 746, MPI_COMM_WORLD, MPI_STATUS_IGNORE);
    ...
```

Execution

```
$ make fifthmessage
$ mpirun -n 5 ./fifthmessage
1: Sent 1.000000 and got 0.000000
2: Sent 4.000000 and got 1.000000
3: Sent 9.000000 and got 4.000000
4: Sent 16.000000 and got 9.000000
0: Sent 0.000000 and got 16.000000
```

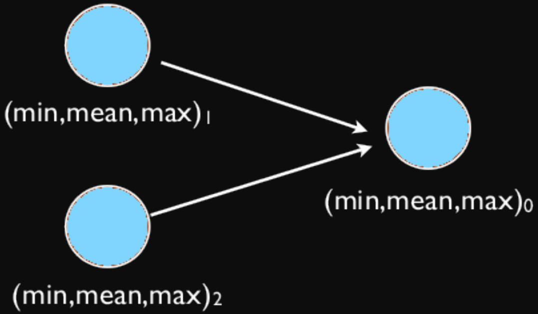
Send/Recv code

```cpp
// fifthmessage.cpp
#include <iostream>
#include <string>
#include <mpi.h>
int main() {
    int rank, size, left, right;
    double msgsent, msgrcvd;
    MPI_Init(nullptr, nullptr);
    MPI_Comm_size(MPI_COMM_WORLD, &size);
    MPI_Comm_rank(MPI_COMM_WORLD, &rank);
    left = rank-1;
    if (left < 0) left = size-1;
    right = rank+1;
    if (right >= size) right = 0;
    msgsent = rank*rank;
    msgrcvd = -999.;
    MPI_Sendrecv(&msgsent, 1, MPI_DOUBLE, right, 749,
                 &msgrcvd, 1, MPI_DOUBLE, left, 749,
                 MPI_COMM_WORLD, MPI_STATUS_IGNORE);
    std::cout << std::to_string(rank) + ": Sent " + std::to_string(msgsent)
              + " and got " + std::to_string(msgrcvd) + "\n";
    MPI_Finalize();
}
```

MPI Reductions

Reductions: Min, Mean, Max Example

- Calculate the min/mean/max of random numbers -1.0 . . . 1.0
- Should trend to -1/0/+1 for a large N.
- How to MPI it?
- Partial results on each node, collect all to node 0.



Reductions: Min, Mean, Max Example (1/2)

```
// Computes the min,mean&max of random numbers
#include <mpi.h>
#include <iostream>
#include <algorithm>
#include <random>
#include <rarray>
int main()
{
  const long nx = 200'000'000;
  // find this process place
  int rank;
  int size;
  MPI_Init(nullptr, nullptr);
  MPI_Comm_size(MPI_COMM_WORLD, &size);
  MPI_Comm_rank(MPI_COMM_WORLD, &rank);
  // determine its subrange of data
  const long nxper=(nx+size-1)/size;
  const long nxstart=nxper*rank;
  const long nxown=(rank<size-1)?nxper
                  :(nx-nxper*(size-1));
  rvector<double> dat(nxown);
  std::uniform_real_distribution<double>
     uniform(-1.0,1.0);
  std::minstd_rand engine(14);
  // each process skip ahead to start
  std::engine.discard(nxstart);
  // compute local data
  for (long i=0;i<nxown;i++)
      dat[i] = uniform(engine);
  const long MIN=0, SUM=1, MAX=2;
  rvector<double> mmm(3);
  mmm = 1e+19, 0, -1e+19;
  for (long i=0;i<nxown;i++) {
      mmm[MIN] = min(dat[i], mmm[MIN]);
      mmm[MAX] = max(dat[i], mmm[MAX]);
      mmm[SUM] += dat[i];
  }
  // send results to a collecting rank
  const long collectorrank = 0;
  if (rank != collectorrank)
    MPI_Ssend(mmm.data(), 3,MPI_DOUBLE,
              collectorrank, 749,
              MPI_COMM_WORLD);
  else {
```

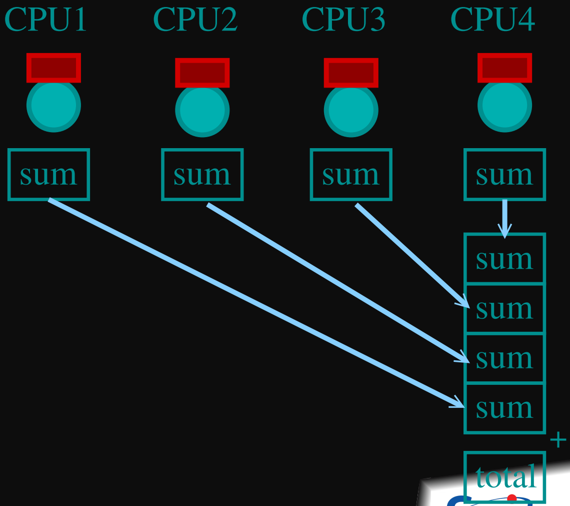
Reductions: Min, Mean, Max Example (1/2)

```
    rvector<double> recvmmm(3);
    for (long i = 1; i < size; i++) {
       MPI_Recv(recvmmm.data(), 3,
                MPI_DOUBLE,
                MPI_ANY_SOURCE, 749,
                MPI_COMM_WORLD,
                MPI_STATUS_IGNORE);
       mmm[MIN] = min(recvmmm[MIN],
                      mmm[MIN]);
       mmm[MAX] = max(recvmmm[MAX],
                      mmm[MAX]);
       mmm[SUM] += recvmmm[SUM];
    }
    // output
    std::cout << "Global Min/mean/max "
         << mmm[MIN] << " "
         << mmm[SUM]/nx <<" "
         << mmm[MAX] << "\n";
  }
  MPI_Finalize();
}
```

Efficiency?

- Requires (P-1) messages
- 2(P-1) if everyone then needs to get the answer.

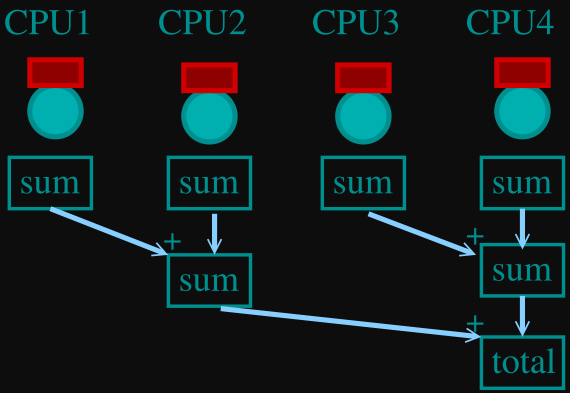
$$T_{comm} = P C_{comm}$$



Better Summing

- Pairs of processors; send partial sums
- Max messages received $\log_2(P)$
- Can repeat to send total back.

$$T_{comm} = 2\log_2(P)C_{comm}$$



Reduction: Works for a variety of operations (+,*,min,max)

MPI Collectives

```
MPI_Allreduce(sendptr, rcvptr, count, MPI_TYPE, MPI_Op, Communicator);

MPI_Reduce(sendbuf, recvbuf, count, MPI_TYPE, MPI_Op, root, Communicator);
```

- `sendptr/rcvptr`: pointers to buffers
- `count`: number of elements in ptrs
- `MPI_TYPE`: one of `MPI_DOUBLE`, `MPI_FLOAT`, `MPI_INT`, `MPI_CHAR`, etc.
- `MPI_Op`: one of `MPI_SUM`, `MPI_PROD`, `MPI_MIN`, `MPI_MAX`.
- `Communicator`: MPI_COMM_WORLD or user created.
- The “`All`” variant sends result back to all processes; non-`All` sends to process `root`.

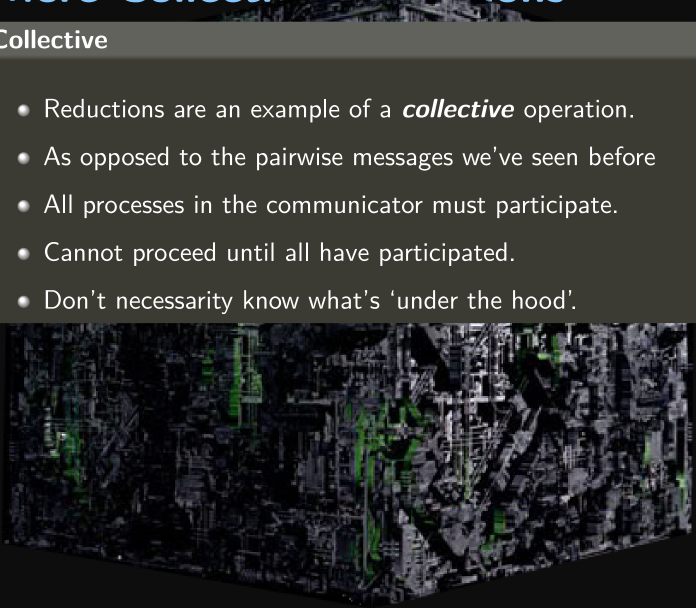
Reductions: Min, Mean, Max with MPI Collectives

```
rvector<double> globalmmm(3);
MPI_Allreduce(&mmm[MIN], &globalmmm[MIN], 1, MPI_DOUBLE, MPI_MIN, MPI_COMM_WORLD);
MPI_Allreduce(&mmm[MAX], &globalmmm[MAX], 1, MPI_DOUBLE, MPI_MAX, MPI_COMM_WORLD);
MPI_Allreduce(&mmm[SUM], &globalmmm[SUM], 1, MPI_DOUBLE, MPI_SUM, MPI_COMM_WORLD);
if (rank==0)
   std::cout << "Global Min/mean/max "
             << mmm[MIN] << " "
             << mmm[SUM]/nx << " "
             << mmm[MAX] << endl;
```

More Collective Operations

Collective

- Reductions are an example of a ***collective*** operation.
- As opposed to the pairwise messages we've seen before
- All processes in the communicator must participate.
- Cannot proceed until all have participated.
- Don't necessarity know what's 'under the hood'.



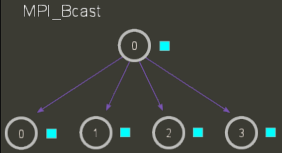
More Collective Operations

Collective

- Reductions are an example of a ***collective*** operation.
- As opposed to the pairwise messages we've seen before
- All processes in the communicator must participate.
- Cannot proceed until all have participated.
- Don't necessarity know what's 'under the hood'.

Other MPI Collectives

Broadcast



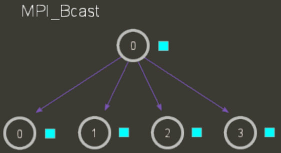
More Collective Operations

Collective

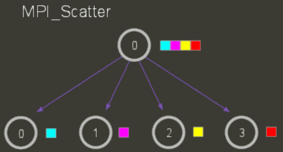
- Reductions are an example of a ***collective*** operation.
- As opposed to the pairwise messages we've seen before
- All processes in the communicator must participate.
- Cannot proceed until all have participated.
- Don't necessarity know what's 'under the hood'.

Other MPI Collectives

Broadcast



Scatter



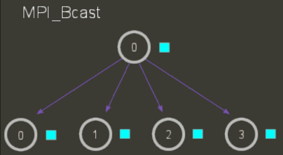
More Collective Operations

Collective

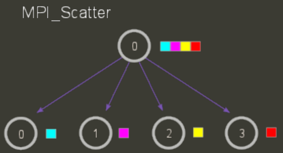
- Reductions are an example of a ***collective*** operation.
- As opposed to the pairwise messages we've seen before
- All processes in the communicator must participate.
- Cannot proceed until all have participated.
- Don't necessarity know what's 'under the hood'.

Other MPI Collectives

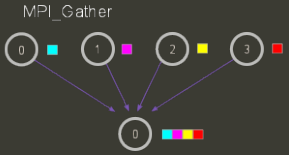
Broadcast



Scatter



Gather



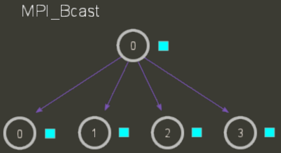
More Collective Operations

Collective

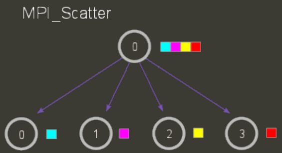
- Reductions are an example of a ***collective*** operation.
- As opposed to the pairwise messages we've seen before
- All processes in the communicator must participate.
- Cannot proceed until all have participated.
- Don't necessarity know what's 'under the hood'.

Other MPI Collectives

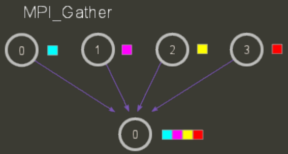
Broadcast



Scatter



Gather



Even more:

- File I/O
- Barriers (avoid!)
- All-to-all . . .

MPI Domain decomposition

Solving the diffusion equation with MPI

Consider a diffusion equation with an explicit **finite-difference**, **time-marching** method.

Imagine the problem is too large to fit in the memory of one node, so we need to do **domain decomposition**, and use **MPI**.

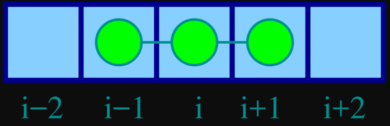
Discretizing Derivatives

- Partial Differential Equations like the diffusion equation

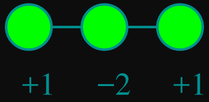
$$\frac{\partial T}{\partial t} = D\frac{\partial^2 T}{\partial x^2}$$

are usually numerically solved by finite differencing the discretized values.

$$\frac{\partial^2 T}{\partial x^2} \approx \frac{T_{i+1} - 2T_i + T_{i-1}}{\Delta x^2}$$



- Implicitly or explicitly involves interpolating data and taking the derivative of the interpolant.



- Larger 'stencils' → More accuracy.

Diffusion equation in higher dimensions

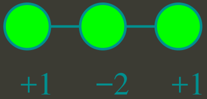
Spatial grid separation: Δx. Time step Δt.

Grid indices: i, j. Time step index: (n)

1D

$$\left.\frac{\partial T}{\partial t}\right|_i \approx \frac{T_i^{(n)} - T_i^{(n-1)}}{\Delta t}$$

$$\left.\frac{\partial^2 T}{\partial x^2}\right|_i \approx \frac{T_{i-1}^{(n)} - 2T_i^{(n)} + T_{i+1}^{(n)}}{\Delta x^2}$$



Diffusion equation in higher dimensions

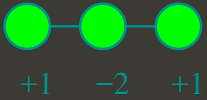
Spatial grid separation: Δx. Time step Δt.

Grid indices: i, j. Time step index: (n)

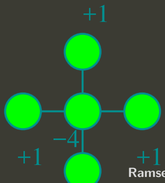
1D

$$\left.\frac{\partial T}{\partial t}\right|_i \approx \frac{T_i^{(n)} - T_i^{(n-1)}}{\Delta t}$$

$$\left.\frac{\partial^2 T}{\partial x^2}\right|_i \approx \frac{T_{i-1}^{(n)} - 2T_i^{(n)} + T_{i+1}^{(n)}}{\Delta x^2}$$



2D



Diffusion equation in higher dimensions

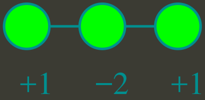
Spatial grid separation: Δx. Time step Δt.

Grid indices: i, j. Time step index: (n)

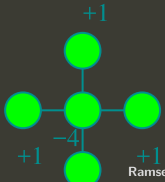
1D

$$\left.\frac{\partial T}{\partial t}\right|_i \approx \frac{T_i^{(n)} - T_i^{(n-1)}}{\Delta t}$$

$$\left.\frac{\partial^2 T}{\partial x^2}\right|_i \approx \frac{T_{i-1}^{(n)} - 2T_i^{(n)} + T_{i+1}^{(n)}}{\Delta x^2}$$



2D



$$\left.\frac{\partial T}{\partial t}\right|_{i,j} \approx \frac{T_{i,j}^{(n)} - T_{i,j}^{(n-1)}}{\Delta t}$$

$$\left.\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right)\right|_{i,j} \approx \frac{T_{i-1,j}^{(n)} + T_{i,j-1}^{(n)} - 4T_{i,j}^{(n)} + T_{i+1,j}^{(n)} + T_{i,j+1}^{(n)}}{\Delta x^2}$$

Stencils and Boundaries

- How do you deal with boundaries?
- The stencil juts out, you need info on cells beyond those you're updating.
- Common solution: ***Guard cells***:
  - Pad domain with these guard celss so that stencil works even for the first point in domain.
  - Fill guard cells with values such that the required boundary conditions are met.

1D

- Number of guard cells $n_g = 1$
- Loop from $i = n_g..N - 2n_g$.

Stencils and Boundaries

- How do you deal with boundaries?

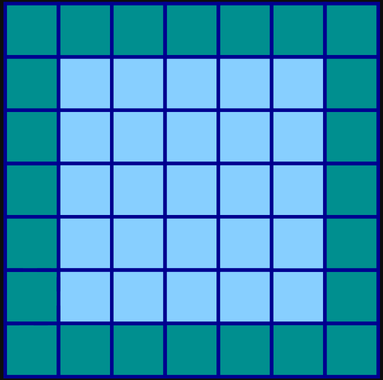
- The stencil juts out, you need info on cells beyond those you're updating.

- Common solution: ***Guard cells***:
  - Pad domain with these guard celss so that stencil works even for the first point in domain.
  - Fill guard cells with values such that the required boundary conditions are met.

1D

- Number of guard cells $n_g = 1$
- Loop from $i = n_g..N - 2n_g$.

2D



Domain decomposition

- A very common approach to parallelizing on distributed memory computers.

Domain decomposition

- A very common approach to parallelizing on distributed memory computers.
- Subdivide the domain into contiguous subdomains.

Domain decomposition

- A very common approach to parallelizing on distributed memory computers.
- Subdivide the domain into contiguous subdomains.
- Give each subdomain to a different MPI process.

Domain decomposition

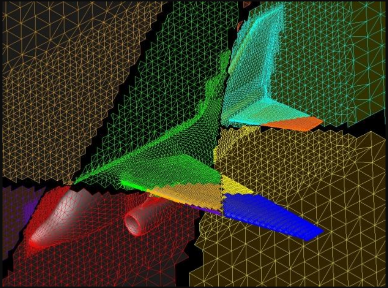
- A very common approach to parallelizing on distributed memory computers.
- Subdivide the domain into contiguous subdomains.
- Give each subdomain to a different MPI process.
- No process contains the full data!

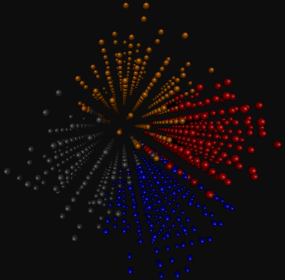
Domain decomposition

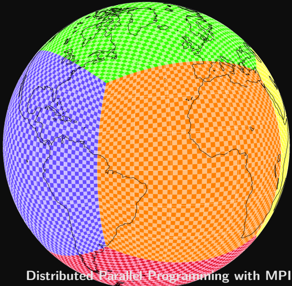
- A very common approach to parallelizing on distributed memory computers.
- Subdivide the domain into contiguous subdomains.
- Give each subdomain to a different MPI process.
- No process contains the full data!
- Maintains locality.

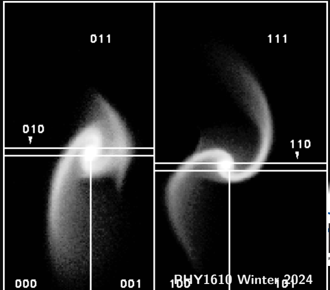
Domain decomposition

- A very common approach to parallelizing on distributed memory computers.
- Subdivide the domain into contiguous subdomains.
- Give each subdomain to a different MPI process.
- No process contains the full data!
- Maintains locality.
- Need mostly local data, ie., only data at the boundary of each subdomain will need to be sent between processes.



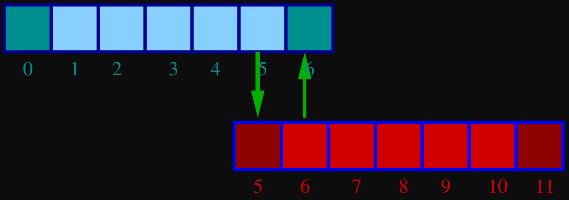






Guard cell exchange

- In the domain decomposition, the stencils will jut out into a neighbouring subdomain.
- Much like the boundary condition.
- One uses guard cells for domain decomposition too.
- If we managed to fill the guard cell with values from neighbouring domains, we can treat each coupled subdomain as an isolated domain with changing boundary conditions.



- Could use even/odd trick, or sendrecv.

1D diffusion with MPI

Before MPI

```
a = 0.25*dt/pow(dx,2);
guardleft = 0;
guardright = n+1;
for (int t=0;t<maxt;t++) {
 T[guardleft] = 0.0;
 T[guardright] = 0.0;
 for (int i=1; i<=n; i++)
   newT[i] = T[i] + a*(T[i+1]+T[i-1]-2*T[i]);
 for (int i=1; i<=n; i++)
   T[i] = newT[i];
}
```

1D diffusion with MPI

Before MPI

```
a = 0.25*dt/pow(dx,2);
guardleft = 0;
guardright = n+1;
for (int t=0;t<maxt;t++) {
 T[guardleft] = 0.0;
 T[guardright] = 0.0;
 for (int i=1; i<=n; i++)
   newT[i] = T[i] + a*(T[i+1]+T[i-1]-2*T[i]);
 for (int i=1; i<=n; i++)
   T[i] = newT[i];
}
```

After MPI

```
MPI_Init(&argc,&argv);
MPI_Comm_rank(MPI_COMM_WORLD,&rank);
MPI_Comm_size(MPI_COMM_WORLD,&size);
left  = rank-1; if(left<0)left=MPI_PROC_NULL;
right = rank+1; if(right>=size)right=MPI_PROC_NULL;
localn = n/size;
a = 0.25*dt/pow(dx,2);
guardleft = 0;
guardright = localn+1;
for (int t=0;t<maxt;t++) {
 MPI_Sendrecv(&T[1],         1,MPI_DOUBLE,left, 11,
              &T[guardright],1,MPI_DOUBLE,right,11,
              MPI_COMM_WORLD,MPI_STATUS_IGNORE);
 MPI_Sendrecv(&T[nlocal],    1,MPI_DOUBLE,right,11,
              &T[guardleft], 1,MPI_DOUBLE,left, 11,
              MPI_COMM_WORLD,MPI_STATUS_IGNORE);
 if (rank==0) T[guardleft] = 0.0;
 if (rank==size-1) T[guardright] = 0.0;
 for (int i=1; i<=localn; i++)
   newT[i] = T[i] + a*(T[i+1]+T[i-1]-2*T[i]);
 for (int i=1; i<=n; i++)
   T[i] = newT[i];
```

1D diffusion with MPI

Before MPI

```
a = 0.25*dt/pow(dx,2);
guardleft = 0;
guardright = n+1;
for (int t=0;t<maxt;t++) {
 T[guardleft] = 0.0;
 T[guardright] = 0.0;
 for (int i=1; i<=n; i++)
   newT[i] = T[i] + a*(T[i+1]+T[i-1]-2*T[i]);
 for (int i=1; i<=n; i++)
   T[i] = newT[i];
}
```

Note:

- the for-loop over `i` could also have been a call to dgemv for a submatrix.
- the for-loop over `i` could also easily be parallelized with OpenMP
- (→ hybrid MPI OpenMP code)

After MPI

```
MPI_Init(&argc,&argv);
MPI_Comm_rank(MPI_COMM_WORLD,&rank);
MPI_Comm_size(MPI_COMM_WORLD,&size);
left  = rank-1; if(left<0)left=MPI_PROC_NULL;
right = rank+1; if(right>=size)right=MPI_PROC_NULL;
localn = n/size;
a = 0.25*dt/pow(dx,2);
guardleft = 0;
guardright = localn+1;
for (int t=0;t<maxt;t++) {
 MPI_Sendrecv(&T[1],         1,MPI_DOUBLE,left, 11,
              &T[guardright],1,MPI_DOUBLE,right,11,
              MPI_COMM_WORLD,MPI_STATUS_IGNORE);
 MPI_Sendrecv(&T[nlocal],    1,MPI_DOUBLE,right,11,
              &T[guardleft], 1,MPI_DOUBLE,left, 11,
              MPI_COMM_WORLD,MPI_STATUS_IGNORE);
 if (rank==0) T[guardleft] = 0.0;
 if (rank==size-1) T[guardright] = 0.0;
 for (int i=1; i<=localn; i++)
   newT[i] = T[i] + a*(T[i+1]+T[i-1]-2*T[i]);
 for (int i=1; i<=n; i++)
   T[i] = newT[i];
}
```

2D diffusion with MPI

How to divide the work in 2d?

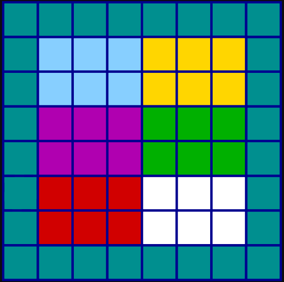
2D diffusion with MPI

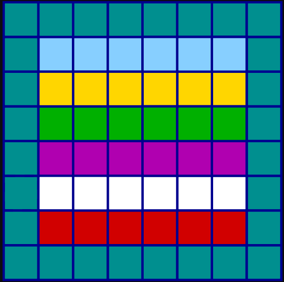
How to divide the work in 2d?

- Less communication (18 edges).
- Harder to program, non-contiguous data to send, left, right, up and down.

2D diffusion with MPI

How to divide the work in 2d?





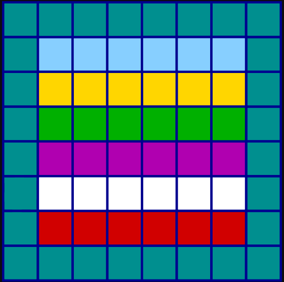
- Less communication (18 edges).
- Harder to program, non-contiguous data to send, left, right, up and down.

- Easier to code, similar to 1d, but with contiguous guard cells to send up and down.
- More communication (30 edges).

Let’s look at the easiest domain decomposition.

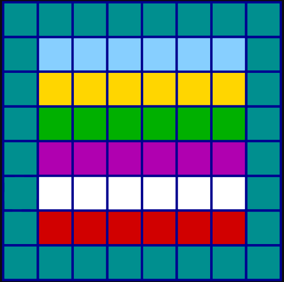
Let’s look at the easiest domain decomposition.

Serial:

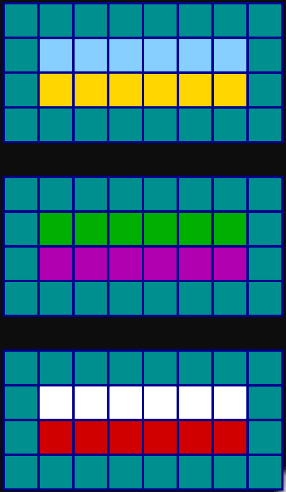


Let’s look at the easiest domain decomposition.

Serial:

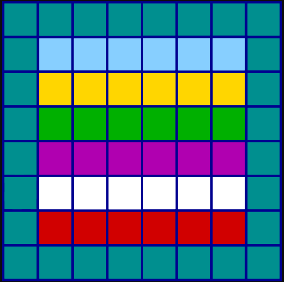


Parallel $(P = 3)$:

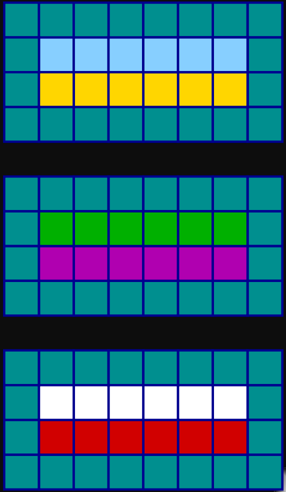


Let’s look at the easiest domain decomposition.

***Serial*:**



***Parallel* $(P = 3)$:**



Communication pattern:

- Copy upper stripe to upper neighbour bottom guard cell.
- Copy lower stripe to lower neighbout top guard cell.
- Contiguous cells: can use count in `MPI_Sendrecv`.
- Similar to 1d diffusion.